RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2025

SECOND YEAR (BATCH 2023-25)

Date : 13/05/2025 **MATHEMATICS**

Time : 11.00 am – 1.00 pm **Paper : CC 19** Full Marks : 50

Answer **all** the questions, maximum you can score is 50.

1. Suppose D is an infinite set. Prove that
   a) If p is a free ultrafilter on D and if $x \in D$ then $\{x\} \notin p$.
   b) If p is a free ultrafilter on D and if A is a finite subset of D then $A \notin p$.
   c) If A is an inifinite subset of D then there is a free ultrafilter p on D such that $A \in p$. [3+4+3]
2. Prove that $(\beta\mathbb{N},+)$ is a semigroup. [8]
3. Find c many free ultrafilters on $\mathbb{N}$. [5]
4. State and prove Ellis's theorem. [1+6]
5. Let $(G,\cdot)$ be a compact left topological semigroup. If $K(G)=\bigcup\{R : R$ is a minimal right ideal of $G\}$ then prove that *K(G)* is the smallest two sided ideal of *G*. [7]
6. Suppose $A \subseteq \mathbb{N}$. Prove that *A* is a additively syndetic *iff* $\bigcup_{t\in F}(A-t)=\mathbb{N}$ for some finite subset *F* of $\mathbb{N}$. [6]
7. Define an additively thick set in $\mathbb{N}$ and give an example. Show that any additively thick set in $\mathbb{N}$ is additively central. [2+5]
8. Let $A \subseteq \mathbb{N}, m \in \mathbb{N}$ and $p \in \beta\mathbb{N}$. Prove that
   (a) $p \in \overline{A-m}$ iff $p+m \in \overline{A}$
   (b) $\overline{p+\mathbb{N}} = p+\beta\mathbb{N}$ [4+3]
9. Consider the set *A* of all composite numbers in $\mathbb{N}$. Examine whether *A* is additively thick. [3]

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