RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2025

SECOND YEAR (BATCH 2023-25)

Date : 10/05/2025 **MATHEMATICS**

Time : 11.00 am – 1.00 pm **Paper : CC 18** Full Marks : 50

Answer **all** the questions, maximum you can score 50.

[All the notations and symbols have their usual meaning.]

1. Transform the following partial differential equation to canonical form:
$x^2u_{tt} - 2txu_{tx} + t^2u_{xx} = \left(\frac{x^2}{t}\right)u_t + \left(\frac{t^2}{x}\right)u_x$. [6]

2. Obtain Riemann-Volterra solution of one dimensional wave equation $u_{xx} = \frac{1}{c^2}u_{tt}$ where u and $\frac{\partial u}{\partial n}$ are given on a curve which is not a characteristic curve. [7]

3. Find the applicable solution of $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\varphi^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ by the method of separation of variables. [8]

4. Find the solution of the equation $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0,\ 0 \le r \le a,\ 0 \le \theta \le 2\pi$ in closed integral form where $u(a,\theta) = f(\theta), f$ being continuous function on the boundary of the circle $r = a.$ [7]

5. Solve the following Dirichlet problem:
$u_{xx} + u_{yy} = 0,\ \ 0 < x < 2, 0 < y < 1$
$u(x,0) = 0, 0 < x < 2\ ,\ u(x,1) = e^x, 0 < x < 2$
$u(0,y) = 0, 0 < y < 1\ ,\ u(2,y) = 0, 0 < y < 1$. [6]

6. Let u be a harmonic function in R. If $P(x,y,z)$ be any point in R and S be the surface of a sphere of radius r having its centre at P lying entirely within R then prove that $u(P) = \frac{1}{4\pi r^2}\iint u(Q)\ dS$ where Q is a point on S and the integration is over S. [6]

7. Find the bounded solution $u(x,t), 0 < x < 1, t > 0$ of the boundary value problem $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 1 - e^{-t}, u(x,0) = x$. [7]

8. Deduce 'Helmholtz's First Theorem' for three dimensional wave equation. [7]

9. Obtain the solution of wave equation
$\frac{\partial^2 u}{\partial t^2} = c^2\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2}\right]$ by the method of separation of variables. [6]

—————— × ——————