

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2025

SECOND YEAR (BATCH 2023-25)

Date : 07/05/2025

MATHEMATICS

Time : 11.00 am – 1.00 pm

Paper : CC 17

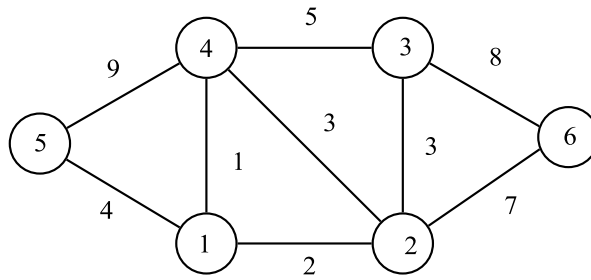
Full Marks : 50

Answer all the questions, maximum you can score 50.

[All the notations and symbols have their usual meaning.]

1. a) Let α be an irrational number and Q be a natural number. Show that there exists a rational number $\frac{p}{q}$ with $1 \leq q \leq Q$ satisfying $\left| \alpha - \frac{p}{q} \right| < \frac{1}{qQ}$.
b) Apply the method of generating functions to solve the following recurrence relation:
 $a_n = 2a_{n-1} - 3^n$ for all $n \in \mathbb{N}; a_0 = 1$. [5+5]
2. a) Is it possible for a simple graph to have a degree sequence (9,8,7,6,5,4,3)? If yes, then provide example. If no, explain why.
b) Prove that in any tree with at least 2 vertices, there are at least two pendant vertices. [3+2]
3. a) Define an induced subgraph by vertex set. Give example.
b) Prove that a simple graph G with p -vertices and satisfying the condition $\delta(G) \geq \frac{p-1}{2}$ is connected, where $\delta(G)$ denotes minimum degree of G .
c) Prove that for a graph G of six vertices either G or its complement contains a triangle. [2+3+3]
4. a) Prove that a graph G is bipartite if and only if all its cycles are of even length.
b) Draw the complement of the graph $K_{2,3}$. [5+2]
5. a) Draw a graph with vertex connectivity 1, edge connectivity 2 and minimum degree 3.
b) Prove that an Euler graph can not have a cut set with an odd number of edges. [4+3]
6. a) Give an example with proper justification of a graph which is Hamiltonian but not Eulerian.
b) Prove that $K_{3,3}$ is non-planar.
c) How do the adjacency matrices of two isomorphic graphs relate each other? [2+3+2]
7. a) Let T be a complete binary tree having p internal vertices. Then prove that T contains $p+1$ leaves.
b) Draw a forest with 8 vertices and 5 edges.
c) Write down the differences between BFS and DFS. [2+2+2]

8. a) What is orientation of a graph?
 b) Sketch two different (non-isomorphic) graphs having same chromatic polynomial.
 c) Using Prim's algorithm, find a minimal spanning tree of the following graph:



[1+3+6]

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