

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. SECOND SEMESTER EXAMINATION, MAY 2025

FIRST YEAR [BATCH 2024-26]

Date : 15/05/2025

MATHEMATICS

Time : 11.00 am – 1:00 pm

PAPER : CC 6

Full Marks : 50

All the symbols have their usual meanings.

Group : - A

Answer all, maximum you can score is 30.

1. Let F be a field and let $p(x) \in F[x]$ be an irreducible polynomial. Show that there exists an extension of F in which $p(x)$ has a root. [5]
2. Determine the degree of the extension $\mathbb{Q}(\sqrt{3+2\sqrt{2}})$ over $\mathbb{Q}$. [3]
3. Given any prime number p and any positive integer n , prove that there exists a unique (upto isomorphism) finite field with p^n elements. [8]
4. Prove that if a field contains the n th roots of unity for n odd then it also contains the $(2n)$ th roots of unity. [3]
5. Determine the galois group of $x^3 - x^2 - 4$. [5]
6. Find a primitive generator for $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{7})$ over $\mathbb{Q}$. [5]
7. Construct a finite field with 8 elements. [5]
8. Determine the minimal polynomial over $\mathbb{Q}$ for $\sqrt{2} + \sqrt{5}$. [2]

Group : - B

Answer all, maximum you can score is 20.

R is a ring with 1 and M is a left R -module.

9. Let $N_1 \subseteq N_2 \subseteq \dots$ be an ascending chain of submodules of M . Prove that $\bigcup_{i=1}^{\infty} N_i$ is a submodule of M . [4]
10. If N is a submodule of M , the annihilator of N in R is $\{r \in R \mid rn = 0 \text{ for all } n \in N\}$. Let M_1 be the abelian group $\frac{\mathbb{Z}}{24\mathbb{Z}} \times \frac{\mathbb{Z}}{15\mathbb{Z}} \times \frac{\mathbb{Z}}{50\mathbb{Z}}$. Find the annihilator of M in $\mathbb{Z}$. [4]
11. Exhibit all $\mathbb{Z}$ -module homomorphisms from $\frac{\mathbb{Z}}{42\mathbb{Z}}$ to $\frac{\mathbb{Z}}{70\mathbb{Z}}$. [5]
12. Let R be a commutative ring. Prove that $\text{Hom}_R(R, R)$ and R are isomorphic as rings. [6]
13. Consider the $\mathbb{Z}$ -module $M_n(\mathbb{Z})$, where $M_n(\mathbb{Z})$ is the set of all $n \times n$ square matrices with entries from $\mathbb{Z}$. Let A and B respectively denote the set of all symmetric matrices and skew-symmetric matrices in $M_n(\mathbb{Z})$. Prove that $\frac{M_n(\mathbb{Z})}{A} \cong B$. [5]

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