# RAMAKRISHNA MISSION VIDYAMANDIRA

**(Residential Autonomous College affiliated to University of Calcutta)**

**M.A./M.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2024**

**FIRST YEAR [BATCH 2024-26]**

Date : 21/12/2024

**MATHEMATICS**

Time : 11 am – 1 pm

**PAPER : CC 4**

Full Marks : 50

*Answer all the questions. Maximum you can score is 50.*

All the symbols have their usual meanings.

1. If $X$ is any non-empty set, then the cardinal number of $X$ is less than the cardinal number of the class of all subsets of $X$. [5]

2. a) If $X=\{a,b,c\}$, let $\tau_1=\{\phi,X,\{a\},\{a,b\}\}$ and $\tau_2=\{\phi,X,\{a\},\{b,c\}\}$ .

   Find the smallest topology on $X$ containing $\tau_1$ and $\tau_2$ , and the largest topology on $X$ contained in $\tau_1$ and $\tau_2$ .

   b) If $X\neq\phi$ , find a basis for $X$ which generates

   i) the discrete topology on $X$.

   ii) the indiscrete topology on $X$. [(2+2)+(3+3)]

3. a) Let $\tau$ and $\tau'$ denoted two topologies on $X$. Show that the identity map $i:(X,\tau)\to(X,\tau')$ is a homeomorphism if and only if $\tau=\tau'$ .

   b) Let $f,g:X\to\mathbb{R}$ be continuous. Show that the set $\{x\in X: f(x)\leq g(x)\}$ is closed in $X$.

   Hence show that the function $h:X\to\mathbb{R}$ , defined by $h(x)=min\{f(x),g(x)\}$ for all $x\in X$, is continuous. [4+(3+3)]

4. a) Show that if $X$ is an infinite set, it is connected in the finite complement topology.

   b) Let $A$ be a proper subset of $X$ and $B$ be a proper subset of $Y$. If $X$ and $Y$ are connected, show that $(X\times Y)\setminus(A\times B)$ is connected. [5+5]

5. a) Show that if $f:X\to Y$ is continuous, where $X$ is compact and $Y$ is Hausdorff, then $f$ is a closed map.

   b) Let $(X,d)$ be a metric space. If $f:X\to X$ satisfies the condition $d(f(x),f(y))=d(x,y)$, for all $x,y\in X$, then $f$ is called an isometry of $X$. Show that if $f$ is an isometery of a compact metric space $X$, then $f$ is bijective and hence a homeomorhism. [4+6]

6. a) Show that any finite $T_1$ space is discrete.

   b) Prove that every compact Hausdorff space is normal. [3+7]

7. Let $p:X\to Y$ be a closed continuous surjective map, such that $p^{-1}(\{y\})$ is compact for each $y\in Y$ . Show that if $X$ is regular, then so is $Y$. [5]

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