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(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2018

SECOND YEAR [BATCH 2016-18]

MATHEMATICS

Date : 10/05/2018

Time : 11 am – 1 pm

Paper : XVIII [PARTIAL DIFFERENTIAL EQUATIONS]

Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

[25 Marks]

Answer any one from Question No. 1 – 2 : [1×13]

1. a) Reduce the following equation $y^2u_{xx}+2xyu_{xy}+x^2u_{yy}-xu_x-yu_y=0$ to its canonical form. [6]

   b) Solve the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ where $0\le x\le a$, $0\le y\le b$; using the conditions $u(0,y,t)=u(x,0,t)=u(a,y,t)=u(x,b,t)=0$, $\left.\frac{\partial u}{\partial t}\right|_{t=0}=0$, $u|_{t=0}=f(x,y)$. [Here a, b, c are positive constants] [7]

2. a) Show that the solution of the equation $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$; $0\le r\le a$, $0\le\theta\le 2\pi$ is $u(r,\theta)=\frac{1}{2\pi}\int_0^{2\pi}\frac{(a^2-r^2)f(\varphi)d\varphi}{a^2-2ar\cos(\varphi-\theta)+r^2}$ where $u(a,\theta)=f(\theta)$, $0\le\theta\le 2\pi$; f being continuous function on the boundary of the circle r = a. [7]

   b) Reduce the equation $u_{xx}+xu_{yy}=0$ to normal form when x > 0. [6]

Answer any two from Question No. 3 – 5 : [2×6]

3. Obtain Riemann-Volterra solution of one dimensional wave equation $u_{xx}=\frac{1}{c^2}u_{tt}$ when $u|_\Gamma$ and $\left.\frac{\partial u}{\partial n}\right|_\Gamma$ are prescribed where Γ is not a characteristic curve.

4. Find the general solution of the following equation $T_{rr}+\frac{1}{r}T_r+\frac{1}{r^2}T_{\theta\theta}+T_{zz}=\frac{1}{\alpha}T_t$ where α is a constant by the method of separation of variables.

5. If the linear second order partial differential equation is of the form Lu = 0 where $L\equiv\sum_{i,j=1}^{n}a_{ij}\frac{\partial^2}{\partial x_i\partial x_j}+\sum_{i=1}^{n}b_i\frac{\partial}{\partial x_i}+c$, $a_{ij}=a_{ji}$, b_i and c are functions of x_1, x_2, ...x_n having continuous second order partial derivatives in $D\subset E^n$ then find the condition for which L will be self adjoint operator.

Group – B

[25 Marks]

Answer any five from Question No. 6 – 13 : [5×5]

6. Show that the function $\omega(x,t)=(1-2xt+t^2)^{-\frac{1}{2}}$ may be expanded in ascending powers of t in the form : $\omega(x,t)=(1-2xt+t^2)^{-\frac{1}{2}}=\sum_{n=0}^{\infty}P_n(x)t^n$ where $P_n(x)$ is Legendre polynomial of degree n.

7. Show that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 \ , & \text{for } m \neq n \\ \dfrac{2}{2n+1} \ , & \text{for } m = n \end{cases}$.

8. Prove that $\int_{-1}^{1} (1-x^2)P_\ell'(x)P_m'(x)dx = \dfrac{2\ell(\ell+1)}{2\ell+1}\delta_{\ell m}$.

9. Show that $\dfrac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$ where $J_n(x)$ is Bessel function.

10. a) Evaluate $\int_0^x x^3 J_0(x)dx$. [2]

    b) Show that $J_n(x+y) = \sum_{K=-\infty}^{\infty} J_K(x)J_{n-K}(y)$. [3]

11. Establish the following integral representation of Hypergeometric function :

$$F(\alpha,\beta,\gamma;x) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}dt \text{ , if } \gamma > \beta > 0.$$

12. Express $(x^3 - 3x^2 + 7x - 2)$ in terms of Hermite polynomials.

13. For Laguerre polynomials, show that $L_n(0) = 1$ and $L_n'(0) = -n$.

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