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(Residential Autonomous College affiliated to University of Calcutta)

M.A. /M.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2018

FIRST YEAR [BATCH 2018-20]

Date : 15/12/2018

MATHEMATICS

Time : 11 am - 1 pm

Paper : V [Complex Analysis]

Full Marks : 50

Answer **any five** questions: [5×10]

1. a) If $f(z)$ is analytic at z_0 with $f'(z_0) \neq 0$, then show that $f(z)$ is one-to-one in some neighbourhood of z_0. [4]

   b) Show that there exists a unique bilinear transformation which maps three given distinct points z_1, z_2 and z_3 in the extended z-plane onto three distinct points w_1, w_2 and w_3 in the extended w-plane respectively. [6]

2. a) State and prove Rouche's theorem. [1 + 4]

   b) Let f be analytic in a domain D and f' is continuous in D. Then show that $\int_C f(z)dz = 0$ for every simple closed contour C, in D. [5]

3. a) Let C be a closed contour and m be a point not on C. Then show that the winding number of C with respect to m is an integer. [5]

   b) State and prove Casorati-Weirstrass theorem. [1 + 4]

4. a) Choosing suitable contour, evaluate $\int_0^{2\pi} \frac{1+\sin\theta}{3+\cos\theta} d\theta$ [6]

   b) Show that the zeros of a non-constant analytic function are isolated. [4]

5. a) Show that $\frac{z}{(z-1)(z-3)} = -3\sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z-1)}$, when $0 < |z-1| < 2$. [4]

   b) Show that if C is the boundary of the triangle with vertices at the points 0, 3i and -4 oriented counter- clock wise then $\left|\int_C (e^z - \bar{z})dz\right| \leq 60$. [4]

   c) Find the Laurent series that represents the function $f(z) = z^2 \sin\frac{1}{z^2}$ in the domain $0 < |z| < \infty$ [2]

6. a) Suppose that the power series $\sum_{n=0}^{\infty} a_n z^n$ has a circle of convergence C of radius $r > 0$. Then show that the series converges to a function $f(z)$ which is continuous at every z in Int(C). [Assume the existence of f] [5]

   b) Let f be an entire function such that Real(f) < 3.79. Then evalute $f'(97+107i)$ [5]

7. a) State and prove Cauchy-Residue theorem. [1 + 4]

b) Show that $\operatorname{Res}_{z=\pi i} \dfrac{z-\sinh z}{z^2 \sinh z} = \dfrac{i}{\pi}$ [5]

8. a) Let C denote the circle $|z| = 1$ taken counter clock-wise direction. Then show that

$\int_C e^{z+\frac{1}{z}}\, dz = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!}$ [4]

b) Let f be a meromorphic function defined on a closed and bounded region R. Show that the number of poles of f in R is finite. [4]

c) State Minimum modulus theorem. [2]

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