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(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2018

FIRST YEAR [BATCH 2018-20]

MATHEMATICS

Date : 12/12/2018

Time : 11 am - 1 pm

Paper : I [Abstract Algebra]

Full Marks : 50

[Answer all the questions. Maximum you can score is 50]

1. a) Prove that there are only two groups of order 6 upto isomorphism.

   b) Describe the dihedral group D_4 and find all subgroups of order 4 of D_4. [6 + 6]

2. a) Can you write Q_8, the quaternion group of order 8 as a direct product of two proper subgroups? Justify your answer.

   b) If G is a finite p-group with $|G|>1$, then prove that $Z(G)$, the centre of G has more than one element.

   c) State and prove Sylow's 3rd theorem. [2 + 3 + 6]

3. a) Suppose G is a group of order 96. Show that G has a normal subgroup of order 16 or 32.

   b) Let G be a group of order pq where p and q are primes and $p>q$. If q does not divide $p-1$ then show that G is cyclic. [5 + 5]

4. a) Let $f:\mathbb{R}\to\mathbb{R}$ be a field isomorphism. Show that $f(x)=x\,\forall x\in\mathbb{R}$.

   b) In the ring $\mathbb{Z}_8[x]$, show that $[4]x+[1]$ and $[4]x+[3]$ are units. [5 + 3]

5. a) Show that every Euclidean domain is a principal ideal domain.

   b) In the integral domain $\mathbb{Z}\left[i\sqrt{5}\right]=\left\{a+ib\sqrt{5}:a,b\in\mathbb{Z}\right\}$, show that 3 is not prime.

   c) Prove that every principal ideal domain satisfies ACCP (ascending chain condition for principal ideals) [5 + 2 + 3]

6. a) Suppose $C([0,1])$ denotes the ring of all real valued continuous functions under pointwise addition and multiplication defined as

   $$\left.\begin{array}{ll}(f+g)(x)=f(x)+g(x), & x\in[0,1]\\ (f.g)(x)=f(x).g(x), & x\in[0,1]\end{array}\right\} f,g\in C([0,1])$$

   Let $M_c=\left\{f\in C([0,1]):f(c)=0\right\}$ where $c\in[0,1]$. Show that M_c is a maximal ideal of $C([0,1])$.

   b) Let R be a Euclidean Domain. Prove that any two elements $a,b\in R$ have a g.c.d.

   c) Prove that a homomorphic image of a left Noetherian ring is left Noetherian. [3 +4 +2]

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