RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2017

SECOND YEAR [BATCH 2015-17]

MATHEMATICS

Date : 08/05/2017

Time : 11 am – 1 pm

Paper : XVIII [Partial Differential Equations]

Full Marks : 50

[Use a separate Answer Book for <u>each group</u>]

[Symbols and notations have their usual meanings]

<u>Group – A</u>

Answer <u>any three</u> questions from <u>Question Nos. 1 to 5</u> : [3×7]

1. Reduce the equation $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ to canonical form and hence solve it.

2. Find Lagrange's Identity for $Lu \equiv \sum_{i,j=1}^{n} a_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial u}{\partial x_i} + cu = 0$ where $a_{ij} = a_{ji}$ and a_{ij}, b_i, c are functions of x_1, x_2, ...x_n having continuous second order partial derivatives in $D \subset E^n$. Also find the condition for which L will be self adjoint.

3. Find Riemann-Volterra solution of $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ where u and $\frac{\partial u}{\partial n}$ are given on a curve which is not a characteristic curve.

4. Obtain Poisson's integral formula $u(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi} \frac{(a^2 - r^2)f(\varphi)}{[a^2 - 2ar\cos(\varphi - \theta) + r^2]}d\varphi$ as the solution of the following interior Dirichlet problem for a circle : $\nabla^2 u = 0, 0 \le r \le a$, $0 \le \theta \le 2\pi$ with boundary condition $u(a,\theta) = f(\theta)$, $0 \le \theta \le 2\pi$.

5. By the method of separation of variables, solve the heat conduction equation
$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$. [$\alpha$ = constant]

Answer <u>any one</u> question from <u>Question Nos. 6 & 7</u> : [1×4]

6. Find the equation of the integral surface of the equation $2y(u-3)u_x + (2x-u)u_y = y(2x-3)$ which passes through the circle u = 0, $x^2 + y^2 = 2x$.

7. Obtain D'Alembert's solution of the initial value problem :
$c^2u_{xx} = u_{tt}, -\infty < x < \infty, t \ge 0$, $u(x,0) = u_0(x), u_t(x,0) = u_1(x)$
where $u_0(x) \in C^2(-\infty,\infty)$, $u_1(x) \in C^1(-\infty,\infty)$.

Group – B

Answer any five questions from Question Nos. 8 to 15 : [5×5]

8. Show that $P_n(x) = \frac{1}{2^n \underline{|n}} \frac{d^n}{dx^n}[(x^2-1)^n]$, where $P_n(x)$ is a Legendre polynomial of degree n.

9. Prove that $\int_{-1}^{1} P_n(x)dx = \begin{cases} 2, & \text{if } n=0 \\ 0, & \text{if } n \geq 1 \end{cases}$.

10. Establish the recurrence relation : $xJ_n'(x) = xJ_{n-1}(x) - nJ_n(x)$, for Bessel's functions.

11. Prove that $\int_0^b xJ_0(ax)dx = \frac{b}{a}J_1(ab)$.

12. Show that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\sin\theta - n\theta)d\theta$.

13. Express $2x^3 + 2x^2 - x - 3$ in terms of Hermite polynomials.

14. Prove the orthogonality property :

$$\int_0^{\infty} e^{-x} L_m(x) L_n(x)dx = \begin{cases} 0, & \text{if } m \neq n \\ 1, & \text{if } m = n \end{cases}$$

for Laguerre polynomials.

15. Show that $\lim_{a\to\infty} {}_2F_1(1, a; 1; \frac{x}{a}) = e^x$.

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