RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2017

SECOND YEAR [BATCH 2016-18]

MATHEMATICS

Date : 01/12/2017

Time : 11 am – 1 pm

Paper : XIV [Graph Theory and Combinatorics]

Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

Answer any one question from Question No. 1-3 : [1×5]

1. There are n medicine boxes. Any two medicine boxes have at least one type of medicine in common and every kind of medicine is contained in just two medicine boxes. How many kinds of medicines are there? Justify your answer. [5]

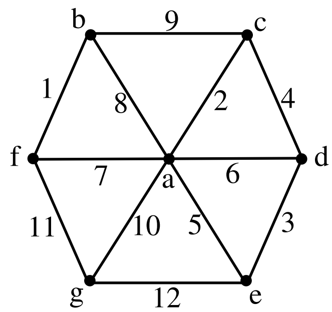
2. a) Define independence number of a graph with example. [2]
   b) How many simple connected graphs can be constructed with n vertices such that the degree of each vertex is even? Justify your answer. [3]

3. Prove that every simple graph with n vertices $(n \geq 2)$ has atleast two vertices of equal degree. [5]

Answer any three questions from Question Nos. 4-8 : [3×10]

4. a) Prove that if a connected planar graph G has exactly n vertices, e edges and f faces then $n-e+f=2$. [5]
   b) Prove that spanning tree of a graph G exists if and only if G is connected. [5]

5. a) Show that a graph G is bipartite if and only if every cycle in G is of even length. [5]
   b) Find a minimum spanning tree of the following graph by applying above algorithm (Step-by-step computation should be shown) [5]

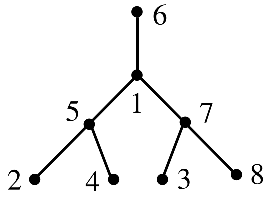


6. a) State and prove five color theorem. [7]
   b) Prove that the number of edges in a bipartite graph with n vertices is atmost $\frac{n^2}{4}$. [3]

7. a) Prove that every tournament contains a directed Hamilton path. [5]
   b) Prove that for any graph G, $\alpha_0(G)+\beta_0(G)=n(G)$, where $\alpha_0(G)=$ vertex-independence number of G, $\beta_0(G)=$ vertex-covering number of G, n(G) = number of vertices in the graph G. [5]

8. a) How many distinct Hamilton cycles are there in a complete graph with n vertices? Justify your answer. [3]

b) Find the Prüfer code of the following tree [3]



c) Draw the tree whose Prüfer code is (6, 4, 1, 1, 5, 3). [4]

<u>Group – B</u>

Answer <u>any three</u> questions from <u>Question Nos. 9-13</u> **:** [3×5]

9. a) Find the number of injective maps from $A = \{10, 11, 12\}$ to $\{2,3,4,5\}$. [2]

b) Find the number of surjective maps from $A = \{a, b, c, d\}$ onto $\{1,2,3\}$. [3]

10. Find the number of square free integers not exceeding 100. [5]

11. Solve the following recurrence relation : $a_n = 7a_{n-1} - 12a_{n-2}$, $n \geq 2$ with the initial conditions $a_0 = 3$, $a_1 = 10$. [5]

12. How many solutions are there of the equation : $x + y + z + w = 20$ in which x, y, z, w are integers, such that x is positive, y is at least 7, z is at least 3 and $w \geq 0$. [5]

13. a) For any integer $n \geq 2$, prove that $\sum_{d|n} \mu(d) = 0$, where the sum is extended over all divisors d of n. [2·5]

b) In how many ways can a total of 16 can be obtained by rolling 4 dice once? [2·5]

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