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(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2017

SECOND YEAR [BATCH 2016-18]

MATHEMATICS

Date : 07/12/2017

Time : 11 am – 1 pm

Paper : XIII [Mathematical Methods]

Full Marks : 50

[Use a separate Answer Book for each group]

Notations and symbols have their usual meanings

Group – A

(***Answer any one question***) [1×20]

1. a) If the function f(x) be Riemann integrable on any finite interval and absolutely integrable on $(-\infty,\infty)$, then prove that $\lim\limits_{t\to\infty}\int_{-\infty}^{\infty} f(x)e^{ixt}dx = 0$. [6]

   b) Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & , \ |x|\le 1\\ 0 & , \ |x|>1\end{cases}$ and hence evaluate $\int_0^{\infty}\frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}dx$. [3+3]

   c) Find the temperature distribution (u) in the semi-infinite rod $0\le x<\infty$ governed by the partial differential equation $\frac{\partial u}{\partial t}=K\frac{\partial^2 u}{\partial x^2}$ subject to the conditions (i) u = 0 when t = 0, $x\ge 0$, (ii) $\frac{\partial u}{\partial x}=-\mu$, a constant, when x = 0 and t > 0. [8]

2. a) If the functions f(x) and g(x) are Riemann integrable on every finite interval and absolutely integrable on $(-\infty,\infty)$, also g(x) is a bounded function on $(-\infty,\infty)$, then prove that $(f\circ g)(x)$ exists for all x in $(-\infty,\infty)$ and $(f\circ g)(x)$ is also bounded on $(-\infty,\infty)$. [6]

   b) If f(t) exists finitely and continuous in the interval on $(-\infty,\infty)$, then prove that $\lim\limits_{|t|\to+\infty} f(t)=0$. Using this result, prove that $F\{f^n(t);s\}=(-is)^n F\{f(t);s\}$, n = 1, 2, 3, ... provided $f', f'',\ldots,f^{n-1}$ are continuous everywhere and absolutely integrable and f^n is piecewise continuous and absolutely integrable in $(-\infty,\infty)$. [3+3]

   c) Solve the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, $0<x<\pi$, $0<y<y_0$ under the boundary conditions $u(0,y)=0$, $u(\pi,y)=1$, $u_y(x,0)=u(x,y_0)=0$. [8]

Group – B

(***Answer any three questions***) [3×5]

3. If $\gamma(x)$ is a good function, prove that the Fourier transform of $\gamma(x)$ is also good function.

4. Prove that $\left\{e^{-\frac{x^2}{n}}\right\}$ and $\left\{e^{-\frac{x^2}{2n}}\right\}$ are regular and equivalent.

5. Prove that $\left\{\sqrt{\frac{n}{\pi}}e^{-nx^2}\right\}$ is regular and defines a generalized function $\delta(x)$. Deduce that the Fourier transform of $\delta(x)$ is 1.

6. Let $\rho(x) = \dfrac{e^{-\frac{1}{1-x^2}}}{\int_{-1}^{1} e^{-\frac{1}{1-t^2}}dt}$ $\quad (|x|<1)$

$= 0 \quad (|x|\geq 1)$

If $f \in K_1$ prove that the sequence $\{\gamma_n(x)\}$, where $\gamma_n(x) = \int_{-\infty}^{\infty} f(u)\rho\{n(u-x)\}ne^{-\frac{u^2}{n}}du$ is regular.

Group – C

(*Answer any three questions*) [3×5]

7. Convert the following differential equation into integral equation : $y''(x)+y(x)=0$ when $y(0) = 0$ and $y'(0)=0$.

8. Obtain the Neumann series of the integral equation : $\phi(x)=1+\lambda\int_0^1 xt\phi(t)dt$ where $0<x<1$.

9. Solve the Abel's integral equation for y(x) : $\int_0^x \frac{y(u)}{\sqrt{x-u}}du = \sqrt{x}$.

10. Solve the integral equation for F(t) : $F(t) = t^2 + \int_0^t F(x)\sin(t-x)dx$.

11. Find the eigen-values of the homogeneous integral equation : $y(x) = \lambda\int_1^2 \left(xt+\frac{1}{xt}\right)y(t)dt$.

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