

# RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2017

FIRST YEAR [BATCH 2017-19]

MATHEMATICS

Paper : V [Complex Analysis]

Date : 23/12/2017

Time : 11 am – 1 pm

Full Marks : 50

Answer **any five** questions :

[5×10]

1. a) Let  $C$  denote the line segment from  $z=i$  to  $z=1$ . Without evaluating the integral, show that  $\left| \int_C \frac{dz}{z^4} \right| < 8$ . [6]  
b) If  $f$  is analytic throughout a given domain  $D$  and  $|f(z)|$  is constant throughout  $D$ , then comment on  $f'(z)$ . [4]
2. a) Prove : Fundamental theorem of algebra (by using Rouché's theorem). [7]  
b) Determine the number of roots of the equation  $z^{19} - 7z^5 + z^3 - 1 = 0$ , inside the circle  $|z|=1$ . [3]
3. a) State and prove Schwarz's lemma. [1+5]  
b) Evaluate  $\int_C z^2 \sin\left(\frac{1}{z}\right) dz$ , where  $C$  is positively oriented circle :  $|z|=1$ . [4]
4. a) Define conformal mapping. [2]  
b) Give an example of a mapping which is isogonal but not conformal. [1]  
c) Show that, a bilinear transformation transforms a circle passing through origin to either a straight line or a circle. [7]
5. a) Show that (by choosing a suitable contour) :  $\int_0^\infty \frac{x^2 dx}{x^4 + a^4} = \frac{\pi}{2\sqrt{2}a}$ , where  $a > 0$ . [7]  
b) If  $z_0$  is a removable singularity of a function  $f$ , then show that  $f$  is analytic and bounded in some deleted neighbourhood of  $z_0$ . [3]
6. a) If  $f$  is an entire function whose real part is bounded, then comment on  $f$ . [3]  
b) State and prove maximum modulus principle. [1+6]
7. a) State and prove Morera's theorem. [1+5]  
b) Find the Laurent series of  $f(z)$  about  $z = -2$ , where  $f(z) = \frac{z}{(z+1)(z+2)}$ . [4]