

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2016

SECOND YEAR [BATCH 2015-17]

MATHEMATICS

Date : 10/12/2016

Time : 11 am – 1 pm

Paper : XV [Operations Research]

Full Marks : 50

Answer any five questions :

[5×10]

1. Consider the following problem

$$\text{Maximize } z = -x_1 + x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 10$$

$$2x_1 + x_2 \leq 11$$

$$x_1 - 2x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

- a) Solve the problem by the simplex method. [5]
b) Suppose that the cost vector $c = (-1, 1)$ is replaced by $(-1, 1) + \lambda(1, 0)$ where $\lambda > 0$. Find the optimal solutions for all values of λ . [5]

2. a) Consider the following problem

$$\text{Maximize } z = 2x_1 + 3x_2 + 5x_3$$

$$\text{Subject to } x_1 + x_2 + 2x_3 \leq 8$$

$$x_1 - x_2 + x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

- i) Find the optimal solution. [3]
ii) Find the new optimal solution if the cost coefficient c_2 changes from 3 to -4 . [2]
b) Solve the following problem by the simplex method for bounded variables

$$\text{Maximize } z = 6x_1 + 4x_2 + 2x_3$$

$$\text{Subject to } 4x_1 - 3x_2 + x_3 \leq 8$$

$$3x_1 + 2x_2 + 4x_3 \leq 10$$

$$0 \leq x_1 \leq 3$$

$$0 \leq x_2 \leq 2$$

$$0 \leq x_3$$

[5]

3. Solve the following problem by Cutting-Plane Algorithm

$$\text{Minimize } z = 4x_1 + 5x_2$$

$$\text{Subject to } x_1 + 4x_2 \geq 5$$

$$3x_1 + 2x_2 \geq 7$$

$$x_1, x_2 \geq 0, \text{ both are integers.}$$

[10]

4. a) Consider the inventory situation in which the stock is replenished uniformly at the rate α . Consumption occurs at the constant rate D . The set up cost is k per order, and holding cost is h per unit item per unit time. If y is the order size and no shortage is allowed, show that

i) The maximum inventory level is $y \left(1 - \frac{D}{\alpha} \right)$.

ii) The total cost per unit time is $TCU(y) = \frac{kD}{y} + \frac{h}{2} \left(1 - \frac{D}{\alpha}\right) y$.

iii) The economic order quantity is $y^* = \sqrt{\frac{2kD}{h \left(1 - \frac{D}{\alpha}\right)}}$, $D < \alpha$ [6]

b) A company can produce an item or buy it from a contractor. If it is produced, it will cost Rs. 20 each time the machines are set up. The production rate is 100 units per day. If it is bought from a contractor, it will cost Rs. 15 each time an order is placed. The cost of maintaining the item in stock, whether bought or produced is Re 0.02 per unit per day. The company's usage of item is estimated at 26,000 unit annually. Assuming no shortage is allowed, should the company buy or produce? [4]

5. A 6-ton vessel can be loaded with one or more of three items. The following table gives the unit weight w_i , in ton and unit revenue in thousand of rupees, r_i , for item i . Obtain the maximum total return. [10]

Item i	w_i	r_i
1	4	70
2	1	20
3	2	40

6. a) Suppose that $f(x)$ has continuous first partial derivatives on a convex set D in R^n . Show that the function $f(x)$ is convex if and only if
 $f(x) + \nabla f(x) \cdot (y - x) \leq f(y)$
for all $x, y \in D$. [6]

b) Let $f(x)$ be a continuously differentiable function on R^n . Show that if $f(x)$ is strictly convex, then $\nabla f(x) = \nabla f(y)$ if and only if $x = y$. [4]

7. a) Suppose that $\{x^{(k)}\}$ is the Newton's method sequence for minimizing a function $f(x)$. If the Hessian $Hf(x^{(k)})$ of $f(x)$ at $x^{(k)}$ is positive definite and $\nabla f(x^{(k)}) \neq 0$, prove that the direction $p^{(k)} = -[Hf(x^{(k)})]^{-1} \nabla f(x^{(k)})$ from $x^{(k)}$ to $x^{(k+1)}$ is a descent direction for $f(x)$. [4]

b) Compute the first two iterates $x^{(1)}, x^{(2)}$ in the Steepest Descent sequence $\{x^{(k)}\}$ for $f(x_1, x_2) = 2x_1^2 + x_2^2 - x_1x_2$ starting with the initial point $x^{(0)} = (1, 4)$. [6]

8. a) Apply the Karush-Kuhn-Tucker Theorem to locate the solution of the following convex programming problem :

$$\text{Maximize } z = 4x_1^2 + 2x_2$$

$$\text{Subject to } 3x_1^2 + 4x_2 \leq 24$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

b) At a public telephone booth arrivals are considered to be Poisson with an average inter-arrival time of 12 minutes. The length of a phone call may be assumed to be distributed exponentially with an average of 4 minutes. Calculate the following :

i) What is the probability that a fresh arrival will not have to wait for the phone? [1]

ii) What is the probability that an arrival will have to wait more than 10 minutes before the phone is free? [2]

iii) What is the average length of queues that form from time to time? [2]

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