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(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2016

SECOND YEAR [BATCH 2014-16]

MATHEMATICS

Date : 02/05/2016

Time : 11 am – 1 pm

Paper : XVI

Full Marks : 50

Unit – I

Answer any two questions **:** [2×17]

1. a) What is a delta-convergent sequence? Show that $\frac{1}{\pi}\frac{\sin \gamma x}{x} \to \delta(x)$ as $\gamma \to \infty$ in $\mathcal{D}'(\mathrm{R}^1)$.

   b) Show that the integral equation, $\varphi(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\varphi(t)dt$ possesses no solution for f(x) = x, but it possesses infinitely many solutions when f(x) = 1.

2. a) If $f \in \mathcal{D}'(\mathrm{R}^n)$ and $a(x) \in C^\infty(\mathrm{R}^n)$, then show that $\frac{\partial}{\partial x_1}(af) = \frac{\partial a}{\partial x_1} f + a\frac{\partial f}{\partial x_1}$.

   b) Find the eigen values and eigen functions of the following integral equation,
   $\varphi(x) = \lambda\int_0^1 (5xt^3 + 4x^2t + 3xt)\varphi(t)dt$.

3. a) Explain how the product of a generalized function $f \in \mathcal{D}'$ with a function $a(x) \in C^\infty$ is defined. Also show that $f(x)\delta(x-a) = f(a)\delta(x-a)$ and $\frac{d}{dx}(\operatorname{sgn} x) = 2\delta(x)$.

   b) Reduce the following differential equation with given initial condition to an integral equation : $y'' - y'\sin x + e^x y = x^2$, y(0) = 2, $y'(0) = -2$.

4. a) Prove the Plemelj's formula, $\lim\limits_{\in \to 0+} \frac{1}{x + i\in} = -\pi i\delta(x) + P\frac{1}{x}$.

   b) Find the solution of the following integral equation, $\varphi(x) = \frac{1}{2}\int_{-1}^{1}(xt + x^2t^2)\varphi(t)dt + x^2$, by the method of resolvent kernel.

Unit – II

Answer any one questions **:** [1×16]

5. a) If f(x) and $f'(x)$ are piecewise continuous functions on every finite interval and if $\int_{-\infty}^{\infty}|f(x)|dx < \infty$ then prove that $f(x) = \frac{1}{\pi}\int_0^\infty\int_{-\infty}^{\infty} f(t)\cos[s(t-x)]dt\,ds$.

   Or,

   State and prove Riemann-Lebesgue's theorem.

   b) Find the Fourier transform of $H(a-|x|)$, where a is any number and $H(\cdot)$ is the Heaviside function.

c) Solve the partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ in the infinite thick slab $S=\{(x,y):-\infty<x<+\infty$ and $0\le y\le a$, a being any positive number$\}$ satisfying the boundary conditions $u(x,0)=f(x)$ and $u(x,a)=g(x)$ for $-\infty<x<+\infty$, where f(x) and g(x) possess continuous derivatives on $(-\infty,\infty)$ and also integrable on $(-\infty,\infty)$.

6. a) State and prove the convolution theorem of Fourier transform. Hence deduce Parseval's theorem.

b) Define finite sine and finite cosine transform.

c) Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$ satisfying the initial and boundary conditions $u(0,t)=0, u(\pi,t)=0, u(x,0)=(0\cdot 1)\sin x+(0\cdot 01)\sin 4x$ and $u_t(x,0)=0$ in the region $0\le x\le \pi,\ t\ge 0$.

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