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(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2016

FIRST YEAR [BATCH 2016-18]

MATHEMATICS

Date : 23/12/2016

Time : 11 am – 1 pm

Paper : IV

Full Marks : 50

<u>Answer all the questions. Maximum you can score is 50.</u>

1. a) Let $\tau = \{A \subseteq \mathbb{N} : n \in A$ implies every positive divisor of n belongs to A$\}$. Show that τ is a topology on $\mathbb{N}$ and is not discrete. [4]

   b) Verify whether the map $f : (\mathbb{R}, \tau) \to (\mathbb{R}, \tau_\ell)$ defined by $f(x) = 1 + 2x^2, x \in \mathbb{R}$ is open and continuous; τ and τ_ℓ denote respectively the usual topology and the cocountable topology on $\mathbb{R}$. [4]

   c) Show that every closed set A in a metric space X is G_δ
   (Hint : verify that $A = \{x \in X : f(x) = 0\}$ for some continuous map $f : X \to \mathbb{R}$). [4]

   d) Describe all compact subsets of $\mathbb{R}$ in cocountable topology. [4]

   e) Prove that $\mathbb{R}$ with usual topology is connected. [4]

   f) Prove that the product of any family of compact spaces is compact. [4]

   g) If Card $X = \alpha$, prove that Card $\mathcal{P}(X) = 2^\alpha$; $\mathcal{P}(X)$ denotes the power set of X. [4]

2. a) Let q be a fixed irrational number. Prove that the set $\{a + bq : a, b \in \mathbb{Z}\}$ is dense in $\mathbb{R}$; $\mathbb{Z}$ is the set of all integers. [5]

   b) Let $\{C_\alpha : \alpha \in \wedge\}$ be a family of connected subsets in a space X such that $C_\alpha \cap C_\beta \neq \phi \ \forall\ \alpha, \beta \in \wedge$. Prove that $\bigcup_{\alpha \in \wedge} C_\alpha$ is also connected. [5]

   c) Prove that $2^{\aleph_0} = c$. [5]

3. a) If $\{A_\lambda : \lambda \in \wedge\}$ is a locally finite family of closed sets in a topological space X whose union is X, show that a function $f : X \to Y$ (Y is any topological space) is continuous iff $f|_{A_\lambda}$ is continuous $\forall\ \lambda \in \wedge$.
   (A family of subsets of a topological space is called locally finite if each point of the space has an open neighbourhood intersecting at most finitely many members of the family) [6]

   b) Prove that a regular Lindelöf space is normal. [6]

   c) Suppose X is a metrizable space such that each uncountable set in X has a limit point. Show that X is separable.
   (Hint : For each $n \in \mathbb{N}$, use Zorn's lemma to produce a set Y_n in X which is maximal with respect to the property that the distance between any two distinct points of Y_n is at least $\frac{1}{n}$). [6]

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