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(Residential Autonomous College under University of Calcutta)

M.A./M.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2014

SECOND YEAR

APPLIED CHEMISTRY

Date : 01/12/2014

Time : 11 am – 1 pm

Paper : XIII

Full Marks : 50

[Use a separate Answer Book for each Group]

Group – A

(Answer any five questions)

1. Find the eigen values and one eigen vector corresponding to any one eigen value of the matrix [5]

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$$

2. Determine the rank of the matrix [5]

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{pmatrix}$$

3. Solve the differential equation : $\frac{d^2y}{dx^2} + 4y = \tan 2x$. [5]

4. Solve : $y^2 + \left(x - \frac{1}{y}\right) \frac{dy}{dx} = 0$. [5]

5. If A and B be two independent events, prove that—

a) $\bar{A}, \bar{B}$ are also independent

b) A and $\bar{B}$ are also independent. [5]

6. There are two identical boxes. The first box contains 5 white, 7 red balls and the second box contains 5 white and 5 red balls. One box is chosen at random and a ball is drawn from it. Find the probability that the ball drawn is white. [5]

7. Find the product of all the values $(1+i)^{4/5}$. [5]

8. Solve the system of linear equations : [5]

$$x + 2y + z = 1$$

$$3x + y + 2z = 3$$

$$x + 7y + 2z = 1$$

Group – B

(Answer any three questions)

9. a) Are the conditions for Rolle's theorem satisfied in case of the following functions defined over the given intervals?

i) $f(x) = |x|$ in $[-1, 1]$

ii) $g(x) = x^2$ in $[-2, 2]$

- b) If $f(x) = (x-a)^m(x-b)^n$ where 'm' and 'n' are positive integers, show that 'c' in the Rolle's theorem divides the segment $a \leq x \leq b$ in the ratio $m : n$. [3+2]

10. Evaluate the following limits :

a) $\lim_{x \rightarrow 0} (\sin x)^{2 \tan x}$

b) $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{1/x}$ [2×2½]

11. a) Evaluate $\iint (x^2 + y^2) dx dy$ over the region bounded by $x \geq 0$, $y \geq 0$ and $x + y \leq 1$.

b) Find the value of $\int_0^{\pi} \frac{x \sin x dx}{1 + \cos^2 x}$. [2+3]

12. a) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$.

b) Prove that $\int_0^{\infty} e^{-x^2} \sqrt{x} dx \times \int_0^{\infty} e^{-x^2} \frac{1}{\sqrt{x}} dx = \frac{\pi}{2\sqrt{2}}$. [2+3]

13. a) Find the divergence and curl of the vector field $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$.

b) Show that $\nabla^2 \left(\frac{1}{r} \right) = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. [3+2]

Group – C

(Answer any two questions)

14. Given the differential equation :

$$\frac{dy}{dx} = x^2 + y, y(0) = 1$$

Determine $y(0.03)$ by Euler's method taking step length $h = 0.01$. [5]

15. Evaluate $\int_0^1 \sqrt{1-x^3} dx$ by

i) Simpson's one-third rule and

ii) Trapezoidal rule, taking six equal sub-intervals, correct to two decimal places. [3+2]

16. Use the following table, to compute $f(0.02)$:

[5]

x :	0.0	0.1	0.2	0.3	0.4
f(x) :	1.0000	1.1052	1.2214	1.3499	1.4918

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