RAMAKRISHNA MISSION VIDYAMANDIRA

Belur Math, Howrah – 711 202

ADMISSION TEST – 2016

MATHEMATICS (Honours)

Date : 15-06-2016 **Full Marks : 50** Time: 11·00 a.m – 12·30 p.m

Instructions for the candidate

Answer all the questions given below. Each question carries **2 marks** for correct answer and **(–1) marks** for wrong answer. Tick (✓) the correct option. The tick must be very clear — if it is smudgy or not clear, no marks will be awarded. **Calculator not allowed.**

1. A set A has 10 elements and let $\mathscr{A} = \{X \subseteq A : |X| \geq 4,\ |X - A| \geq 4\}$ where $|X|, |X - A|$ respectively denote the number of elements in X and the number of elements in X – A. Then the number of elements in $\mathscr{A}$ is

   a) 648 b) 672 c) 692 d) 712

2. A set A contains 3 elements. The number of maps from A to A which are not surjective is

   a) 15 b) 18 c) 21 d) 24

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3, x \in \mathbb{R}$; where $\mathbb{R}$ is the set of all real numbers. Then

   a) f is injective but not surjective b) f is surjective but not injective
   c) f is neither injective nor surjective d) f is bijective

4. Suppose X is a finite set and Φ be the set of all binary relations on X. Suppose Φ contains n relations. Then the possible value of n is

   a) 112 b) 210 c) 386 d) 512

5. Suppose $X = \{1, 2\}$. The number of transitive relations on X is

   a) 7 b) 10 c) 13 d) 15

6. Two tangents, perpendicular to each other, to the parabola $y^2 = 4ax$ intersect on the line

   a) x = a b) x + a = 0 c) x + 2a = 0 d) x – 2a = 0

7. The foci of the ellipse $25(x+1)^2 + 9(y+2)^2 = 225$ are

   a) $(-1, 2), (6, 1)$ b) $(-1, -2), (1, 6)$ c) $(1, -2), (1, -6)$ d) $(-1, 2), (-1, -6)$

8. The equation of the director circle of the hyperbola $9x^2 - 16y^2 = 144$ is

   a) $x^2 + y^2 = 7$ b) $x^2 + y^2 = 9$ c) $x^2 + y^2 = 16$ d) $x^2 + y^2 = 25$

9. The circles $x^2 + y^2 - 4x + 10y + 20 = 0$ and $x^2 + y^2 + 8x - 6y - 24 = 0$

   a) touch each other internally b) touch each other externally
   c) cut each other d) cut each other orthogonally

10. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$, $\vec{c} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ form the sides of a

    a) equilateral triangle b) isosceles triangle c) right angled triangle d) none of these

11. The value of $\lim\limits_{n\to\infty}\left(\frac{n}{n^2} + \frac{n+1}{n^2} + \frac{n+2}{n^2} + \ldots + \frac{2n}{n^2}\right)$ is

    a) 1 b) 0 c) $\frac{1}{2}$ d) $\frac{3}{2}$

12. The jump of $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ at $x = 0$ is

    a) 0 b) –1 c) 2 d) 1

13. At x = 0, the function $f(x) = x|x|$ is

    a) continuous only b) discontinuous
    c) continuous and differentiable d) continuous but not differentiable

14. In the MVT, $f(a+h) = f(a) + hf'(a+\theta h)$, for a = 1, h = 3 and $f(x) = \sqrt{x}$ the value of 12θ will be
    a) 1 b) 11 c) 5 d) 10

15. If $f(x) = \mu x - \sin x$ be a monotonic increasing then
    a) $\mu > -1$ b) $\mu < 1$ c) $\mu > 1$ d) $\mu < -1$

16. A function f is defined by $f(x) = \frac{1}{2^{r-1}}$, $\frac{1}{2^r} < x \le \frac{1}{2^{r-1}}$, r = 1, 2, 3, ... The value of $\int_0^1 f(x)dx$ is
    a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{2}{3}$ d) none of these

17. The area bounded by curves $y = x^2$, $y = \frac{2}{1+x^2}$ is
    a) $\pi - \frac{2}{3}$ b) $\frac{\pi}{2} - \frac{1}{3}$ c) $2\pi - \frac{1}{3}$ d) none of these

18. The sum of the infinite series $\cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + ...$ is equal to
    a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{8}$

19. If $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\tan^{51} x} = K$ then value of K is
    a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{12}$

20. Numerical value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$ is
    a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) none of these

21. Ignoring the order of drawing, two cards are drawn from a full pack of 52 cards. The probability of one is a heart and the other is a diamond is
    a) $\frac{25}{102}$ b) $\frac{13}{102}$ c) $\frac{26}{102}$ d) $\frac{52}{102}$

22. A box contains twenty tickets of identical appearance, the tickets being numbered 1, 2, 3, ..., 20. If 3 tickets are chosen at random, the probability that the numbers on the drawn tickets are in arithmetic progression is
    a) $\frac{3}{38}$ b) $\frac{20}{38}$ c) $\frac{3}{20}$ d) $\frac{9}{20}$

23. A random variable has the following probability distribution

x	:	4	5	6	8
probability	:	0·1	0·3	0·4	0·2

    The standard deviation (S.D) of the random variable is
    a) 1·22 b) 2·22 c) 3·23 d) 4·24

24. The differential equation of all circles each of which touches the axis of x at the origin is
    a) $(x^2 - y^2)\frac{dy}{dx} = 2xy$ b) $(x^2 - y^2)y = 2x\frac{dy}{dx}$ c) $\left(y - \frac{dy}{dx}\right)\frac{dy}{dx} = 2x$ d) $2xy = (x^2 + y^2)\frac{dy}{dx}$

25. The general solution of the differential equation $\frac{dy}{dx} + Py = Q$, can be written in the form
    a) $y = u + v + k$ b) $y = k(u - v) + v$ c) $y = k(u + v) + v$ d) $y = k(u - v) + u$
    where P, Q and K are constant, u and v are its two particular solutions.

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