RAMAKRISHNA MISSION VIDYAMANDIRA

Belur Math, Howrah – 711 202

ADMISSION TEST – 2015

MATHEMATICS (Honours)

Date : 18-06-2015 **Full Marks : 50** Time: 01·00 p.m – 02·30 p.m

Instructions for the candidate

Answer all the questions given below. Each question carries 2 marks for correct answer and –1 for wrong answer. Tick (✓) the correct option. The tick must be very clear — if it is smudgy or not clear, no marks will be awarded. Do not use calculator or mobile.

Name of the student : _____

Application No. : _____

1. If the complex number z is such that $|z-1| \le 1$ and $|z-2| = 1$, then the maximum possible value of $|z|^2$ is

   a) $3\sqrt{3}$ b) 9 c) $\sqrt{3}$ d) 3

2. Coefficient of x^{10} in the expansion of $(1+x^2-x^3)^8$ is

   a) 528 b) 506 c) 476 d) 458

3. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways in which this can be done is

   a) 216 b) 240 c) 600 d) 3125

4. The equation $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$ has

   a) no solution b) only one solution
   c) only two solutions d) more than two solutions

5. x-axis divides the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$ in the ratio

   a) 121 : 6 b) 125 : 6 c) 121 : 4 d) 125 : 4

6. The sum to (n+1) terms of the series $\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + \ldots$ is

   a) $\frac{1}{n+1}$ b) $\frac{1}{n+2}$ c) $\frac{1}{n(n+1)}$ d) none of these

7. The number of divisors of 1029, 1547 and 122 are in

   a) A.P b) G.P c) H.P d) none of these

8. The degree and order of the differential equation of all parabolas whose axis is x-axis are

   a) 2, 1 b) 1, 2 c) 3, 2 d) none of these

9. The points representing the complex number z for which $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$ lie on

   a) a straight line b) a circle c) an ellipse d) a parabola

10. If $A=\begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $(I+A)(1-A)^{-1}$ is equal to

a) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ b) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ c) $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$ d) $\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

11. The minimum value of $f(x)=|3-x|+|2+x|+|5-x|$ is

a) 0 b) 7 c) 8 d) none of these

12. The acute angle between the lines whose direction cosines are given by $\ell+m+n=0$, $\ell^2-m^2+n^2=0$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) none of these

13. The least value of 'a' for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$ has at least one solution in $\left(0,\frac{\pi}{2}\right)$ is

a) 5 b) 3 c) 9 d) 2

14. The unit vector in the direction of $\vec{a}+\vec{a}\times\vec{b}+\vec{b}$ where $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ is

a) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$ b) $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$ c) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ d) none of these

15. The natural domain of definition of the function $f(x)=\sqrt[6]{4^x+8^{\frac{2}{3}(x-2)}-2^{2(x-1)}-52}$ is

a) [–3, 3] b) (–3, 3) c) $(-\infty,-3)$ d) $[3,\infty)$

16. The straight line $x+y=k$ touches the parabola $y=-x^2+x+1$ if

a) k = 1 b) k = 2

c) k = 3 d) k may be assume any value

17. The value of the determinant $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix}$ is

a) $\frac{1}{2}$ b) –3 c) 0 d) none of these

18. Let X contains 4 elements and Y contains 3 elements. The number of non-constant maps from X to Y is

a) 9 b) 24 c) 61 d) 78

19. Let □ and □ denote respectively the set of all natural numbers and the set of all integers. Then

a) there is no bijective map from □ to □

b) there is a unique bijective map from□ to □

c) there are finitely many bijective maps from □ to □

d) there are infinitely many bijective maps from □ to □

20. Let X contains 2 elements. The number of binary relations on X which are not reflexive is

a) 4 b) 8 c) 12 d) 16

21. The sum of two prime numbers is 63. The difference of their squares is

a) 1917 b) 2319 c) 2919 d) 3717

22. Two fair dice are thrown. The probability that the sum of two outcomes is divisible by 4 is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{5}{18}$

23. Let $f(x) = \begin{cases} |\sin x|, & 0 \le x \le 3\pi \\ \sin x, & x > 3\pi \end{cases}$. Then

a) f is differentiable

b) f is not differentiable at finite number of points

c) f is not differentiable at infinitely many points

d) f is not continuous

24. A set X contains 3 elements. The number of maps from X to X which are not surjective is

a) 15 b) 18 c) 21 d) 24

25. Suppose $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is defined by $f((x, y)) = 2^x 3^y$ for all $(x, y) \in \mathbb{N} \times \mathbb{N}$. Then

a) f is a bijective map

b) f is injective but not surjective

c) f is surjective but not injective

d) f is neither injective nor surjective

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FOR ROUGH WORK